Statistical Analysis of the Focus Value in the ESO 2.2 Telescope

Preliminary Report

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Chapter 1

Introduction

The right determination of the Focus Value, whether to start a Focus sequence or to correct the actual one (e.g. due to weather changes) makes an excellent way to save time in an astronomic observation nights. However, due to the great number of determinant factors in its definition, which involves from constructive characteristics to weather and pointing variables; in addition to the great difficulty to define the exact effect of each one of this parameters in the changes of the distance of the mirrors, makes that the alternative of an Statically Analysis of the info stored (for more than a year) in the ESO 2.2 Telescope becomes a good tentative of solution for this point.

The info stored includes, besides the Focus Value (in encoder units), the Long and Short Serrurier Temperature, the Main Mirror Temp., the Sidereal Time of the Focus determination, the Right Ascension and Declination coordinates, the date, the used filter and the Time of the exposure.

This work will target to the study of the dependence between the Focus Value and the Long Serrurier Temperature, with the Main Mirror Temperature, the Hour Angle, Declination and the Exposure Time, in every filter used; and to determine if it's possible to build a multiple variable regression model for the Focus Value. To this it will be followed a statistical system, which consist in the progressive selection of variables which makes better (significantly) the accuracy of the model.

Chapter 2

Preliminary Analysis of the Data

As the Focus Value depends significantly with the used filter (approximately is possible to represent that with a little offset applied to the value), the first done to the data was to separate them between the different filters used during this time. This produces the next summary (Table 1).

Table 1

Filer (ESO number)	N of data
#0	39
#840	2
#841	14
#842	217
#843	65
#844	141
#845	112
#846	4
#848	2
#849	5
#850	8
#851	1
#852	7
#854	5
#856	24
#858	7
#859	32
#873	1
#875	2

Due to the poor number of data in some filters, it was decide to make the analysis just with the #0, #842, #843, #844 and #845 (ESO id) filters.

2.1 First Analysis

According to the selection method of the model it must be determined the variable which, in an independent way, produces the most exact regression model, this one will be called "primary variable". To determine this it was necessary to plot the Focus Value against each one of the variables, determining the type of curve which better fits the distribution of the values (this is

one of the most critical steps in the method). After realized this, the respective coefficients, associated to the curve, were evaluated according to the Multiple Variable Regression techniques (minimum square error values), and the value of the "Multiple Determination Coefficient" (R2), which represents the percentage, in the change of the Focus Value, explained or predicted by the model; this is the value used to determine the accuracy or fitness of the model.

Focus Value against Long Serrurier

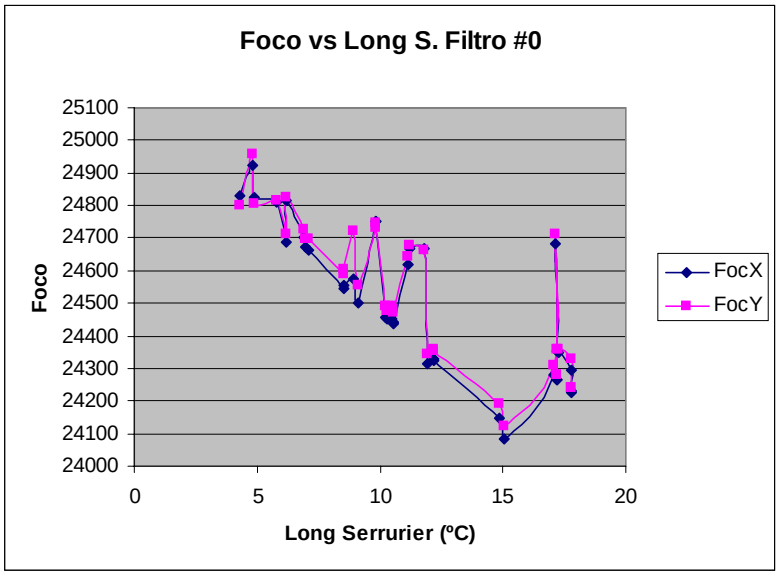


Figure 1

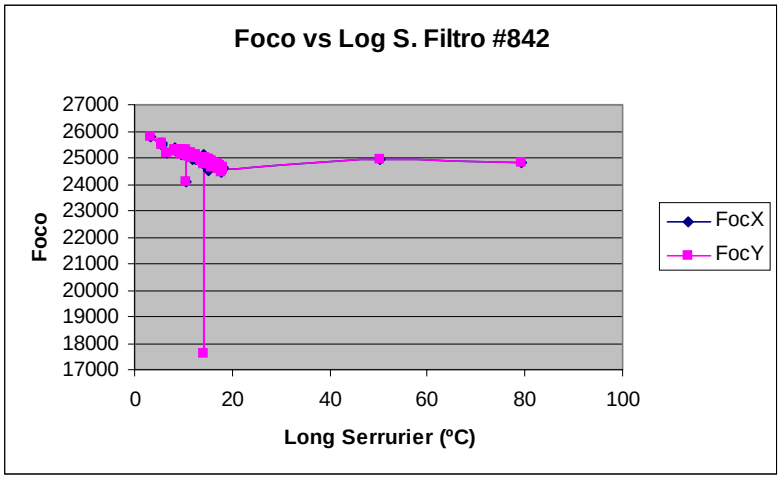


Figure 2

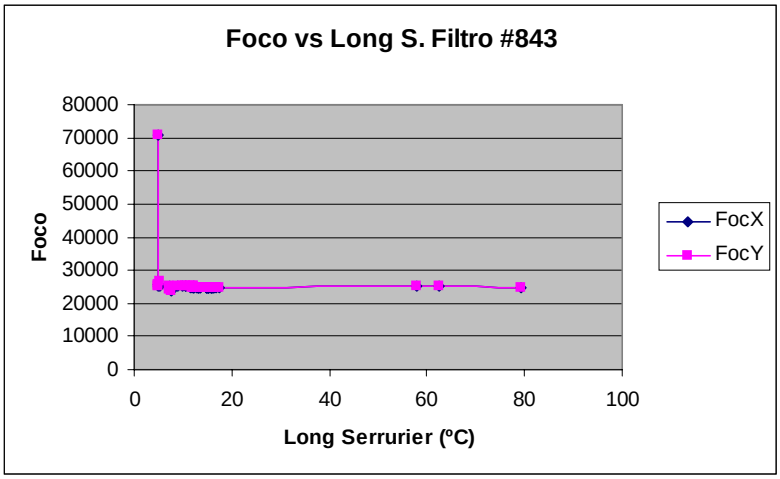


Figure 3

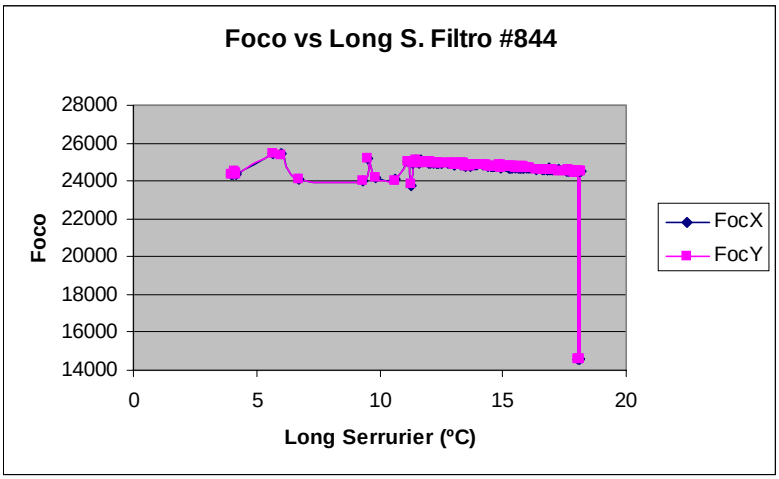


Figure 4

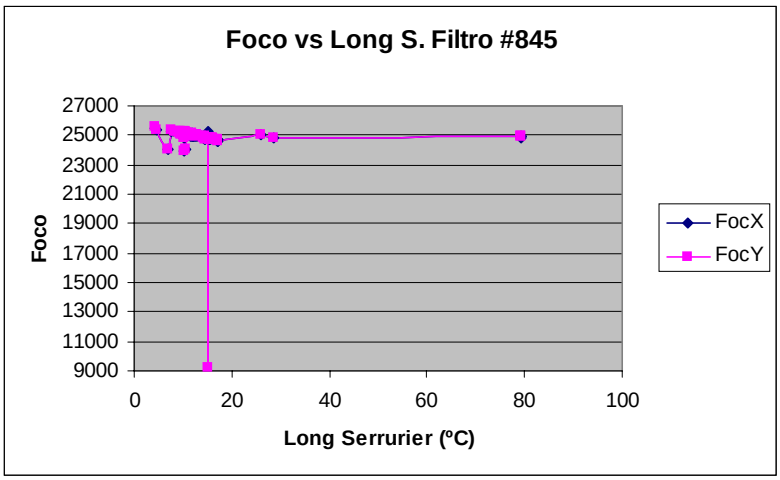


Figure 5

From the observation of Figures 1 to 5, it could be determined some points:

- The values associated to Filter #0 doesn't have a reasonable distribution, which can be due to a multiple filter association with this id number; anywise, these values will not be used.
- For each filter there are a few number of values which are obviously wrong, because they escaped to far from the tendency or are associated to Serrurier Temperatures too high (more than 50 C, for example). It obliged to "clean" the tables, taking care of minimizes this "erasing of values".

The result of these is presented in figures 6 to 9 (in which appears the better curve associated to the distribution, obtained with the MatLab application).

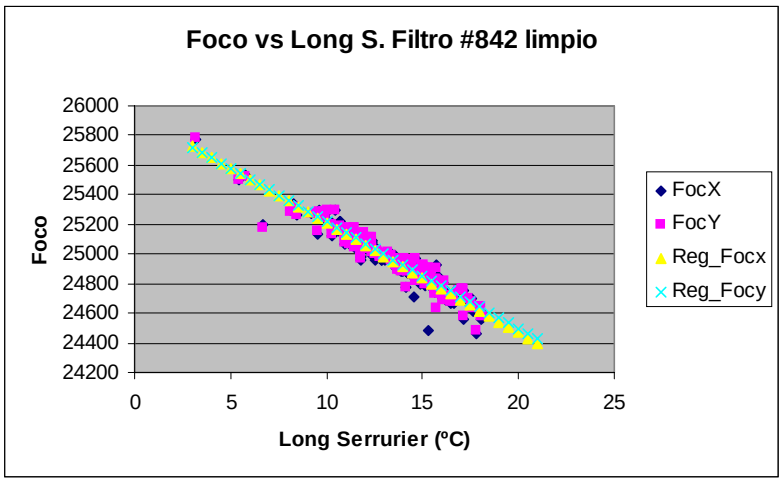


Figure 6

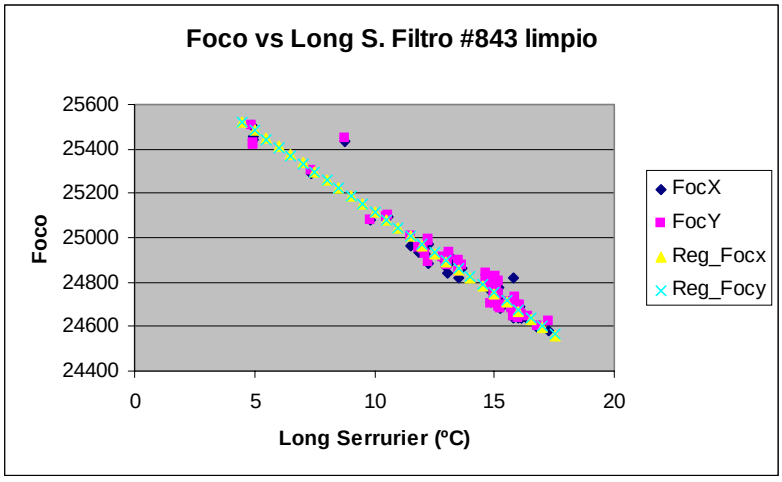


Figure 7

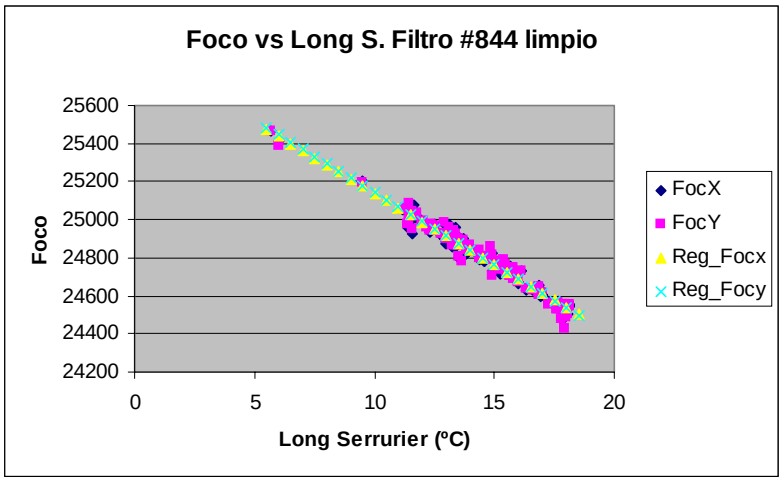


Figure 8

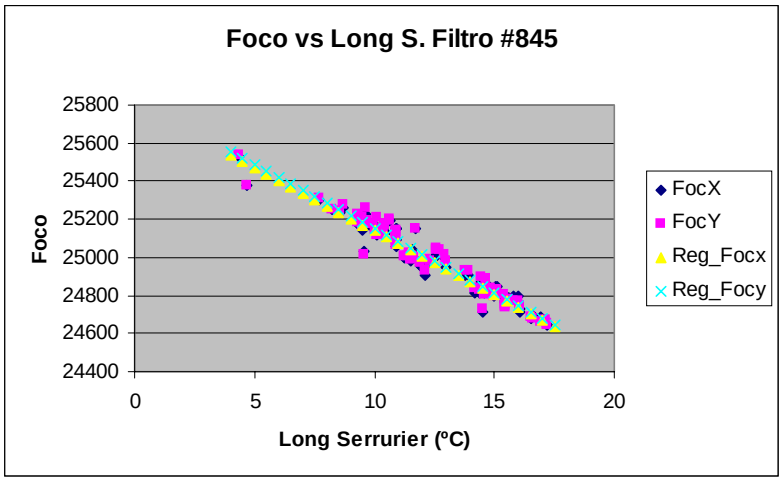


Figure 9

From these images it can be determined, with good accuracy, the linear dependence of the Focus Value with the Long Serrurier Temp., which is not surprising, because in that way was worked until now in the 2.2 ESO Telescope.

That is, the model associated to the Focus Value has the form:

$Y=\beta_0 + \beta_1 * X + \varepsilon$

Where Y is the Focus Value, X is the Long Serrurier Temperature, β_0 and β_1 are the estimation coefficients and ε is the error of the estimation.

Working with minimum square errors matricial Notetion, the estimators for β_0 and β_1, the variance value (s^2) and the Coefficient of Multiple Determination (R^2), for each of the filters, are:

Filter #842:

Note: (β=[β_0 β_1])

Focus X	Focus Y
β = [25.947 -74]	β = [25.936 -72]
s^2 = 2877	s^2 = 2372,9
R^2 = 0,9256	R^2 = 0,9354

Filter #843:

Focus X	Focus Y
β = [25.855 -74]	β = [25.846 -73]
s^2 = 2207	s^2 = 2221
R^2 = 0,9805	R^2 = 0,9801

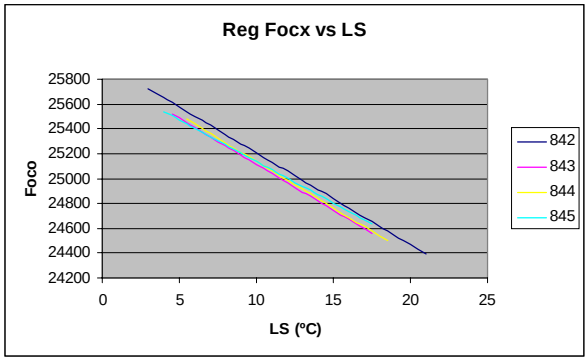
Filter #844:

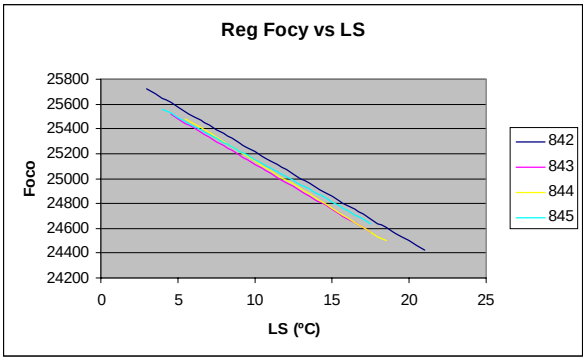
Focus X	Focus Y
β = [25.888 -75]	β = [25.902 -76]
s^2 = 862,08	s^2 = 1053,3
R^2 = 0,9852	R^2 = 0,9822

Filter #845:

Focus X	Focus Y
β = [25808 -67]	β = [25.831 -68]
s^2 = 1745,6	s^2 = 5448,8
R^2 = 0,9798	R^2 = 0,9777

As it can be observed, the accuracy of the models in these filters is very good; in figures 10 and 11 it can be observed these curves and its respective rates.

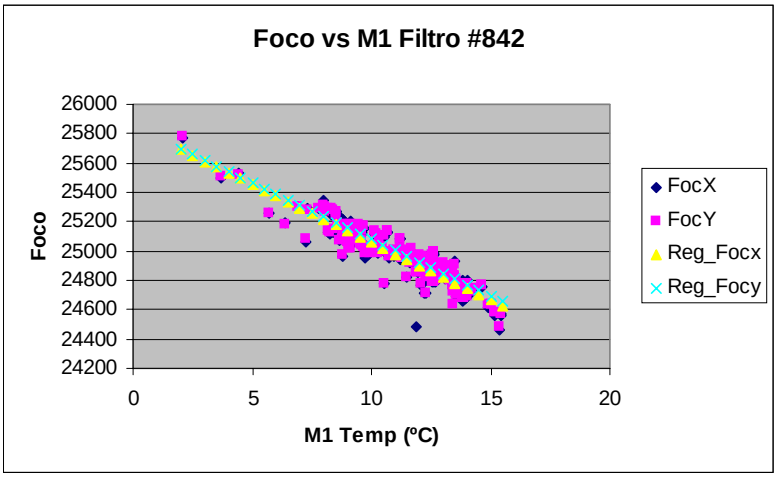


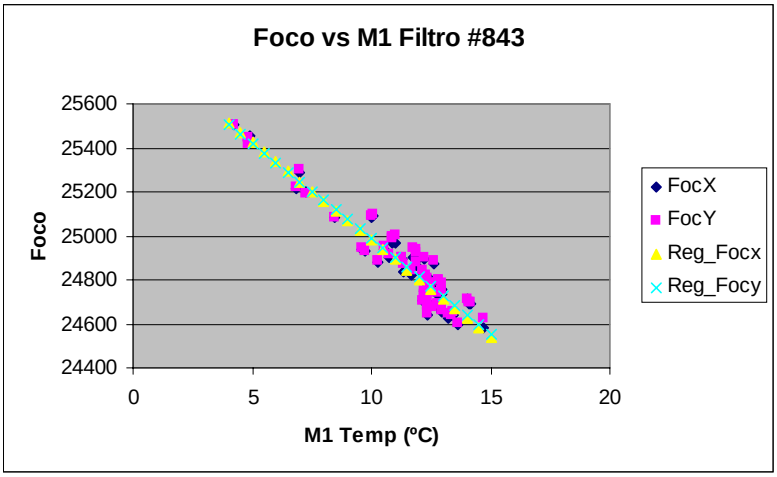


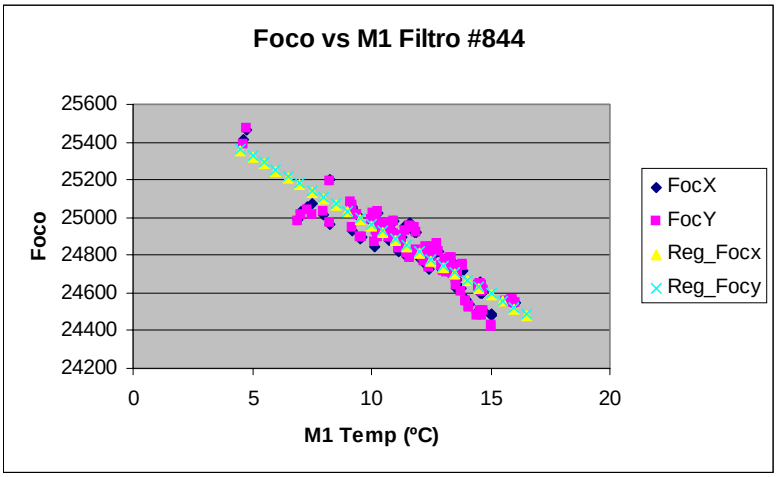
Figures 10 and 11

Focus Value against Main Mirror Temperatures

In a similar way was carried out a first analysis of the Focus data values, now against the Main Mirror Temperature (M1). After a first cleaning of the data, the plots obtained for each filter is presented in figures 12 to 15, with its respective associated curves.







Figures 12, 13 and 14

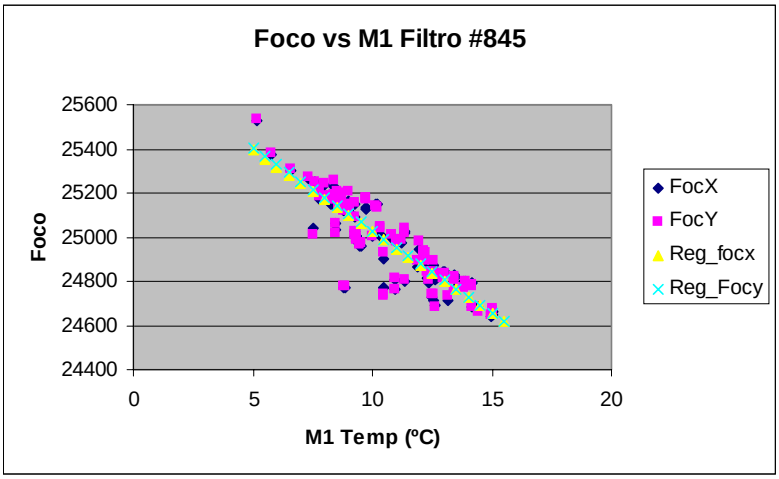


Figure 15

Again, it can be estimated with relative security that the dependence of the Focus Value with this variable is linear, i.e. in the form of

$Y=\beta_0 + \beta_1 * X + \varepsilon$

The results for this variable are:

Tables FocvsM1
Filter #842:

Focus X	Focus Y
β = [25849 -79]	β = [25.849 -77]
s^2 = 5448,8	s^2 = 4685,3
R^2 = 0,853	R^2 = 0,8667

Filter #843:

Focus X	Focus Y
β = [25870 -89]	β = [25.856 -87]
s^2 = 4038,9	s^2 = 4549,9
R^2 = 0,9624	R^2 = 0,9570

Filter #844:

Focus X	Focus Y
β = [25685 -73]	β = [25.692 -73]
s^2 = 3621,6	s^2 = 4321,0
R^2 = 0,9344	R^2 = 0,9230

Filter #845:

Focus X	Focus Y
β = [25763 -74]	β = [25.779 -75]
s^2 = 7458,2	s^2 = 8262,4
R^2 = 0,8976	R^2 = 0,8887

As it can be appreciated, the values of the R^2 coefficient are, in these cases, lower than the which ones obtained for the Long Serrurier Temperature, so the last one stills being the better candidate to primary variable of the definitive model; but however these values are pretty good for some filters.

Focus Value Against Hour Angle

The next used variable was the Hour Angle, calculated over the difference between the Sidereal Time and the Right Ascension. The results of the pre-cleaning process and the respective plotting is showed in figures 16 to 19.

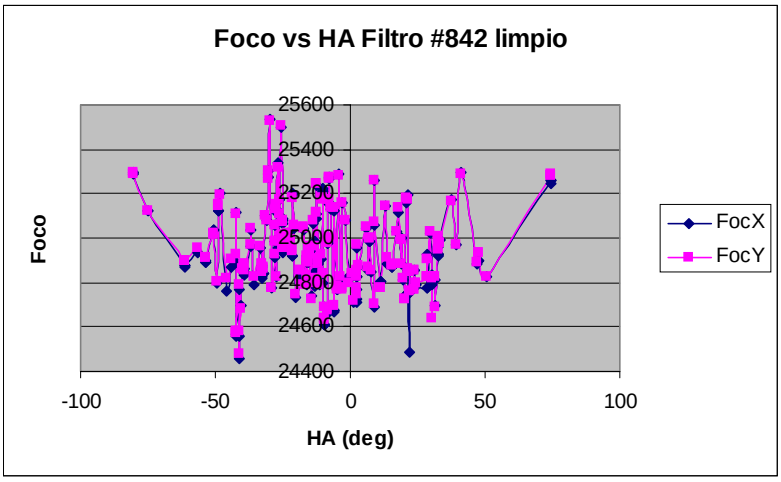


Figure 16

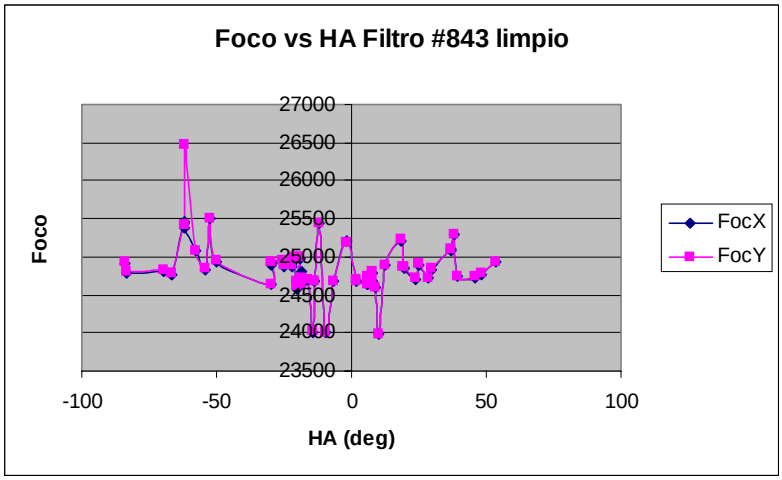


Figure 17

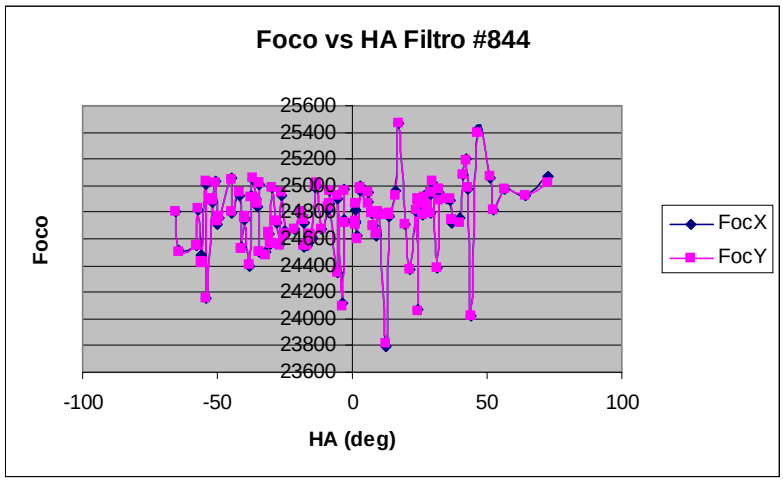


Figure 18

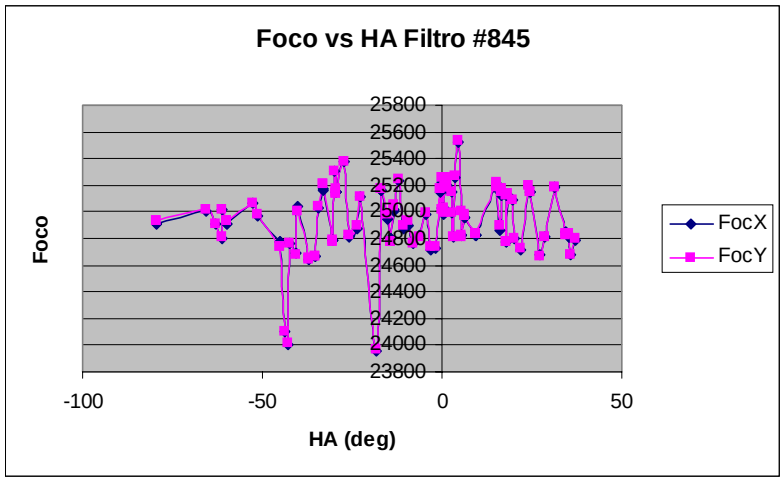


Figure 19

As it can be watched, in this case the dependence is not easy to determine, however, it shows a little concave form, which can be associated with a relationship of second order (or greater, with the rest of lower orders coefficients zero). It was determined to try to determine a second order model, because if it fails, the respective coefficients will be zero. The results are showed subsequently:

Filter #842:

Note: $\beta=[\beta_0\ \beta_1\ \beta_2]$

mod: $Y=\beta_0 + \beta_1 \cdot X + \beta_2 \cdot X^2$

Focus X	Focus Y
β = [24906 0 0]	β = [24915 0 0]
S^2 = 32761	s^2 = 30650
R^2 = 0,0385	R^2 = 0,046

Filter #843:

Focus X	Focus Y
β = [24915 –1 0]	β = [24766 0 0]
S^2 = 78798	s^2 = 10987
R^2 = 0,1072	R^2 = 0,1268

Filter #844:

Focus X	Focus Y
β = [24722 2 0]	β = [24729 2 0]
S^2 = 52373	s^2 = 53674
R^2 = 0,0762	R^2 = 0,0532

Filter #845:

Focus X	Focus Y
β = [24997 1 0]	β = [24990 1 0]
S^2 = 59125	s^2 = 57762
R^2 = 0,2693	R^2 = 0,2741

Statically, these models are not valid neither dependable because its bad correlation R. However, model of higher orders neither will be useful, because its coefficients will annul itself and, in case that doesn't happen, the model will be too much complex in its development and the improvements obtained won't be significant. By another side, the form of this curve is symmetric, so a model $Y=X^n$ could be recommended; however, plotting these values in logarithmic scale (which gives a straight line with a rate of value n) produces a constant value curve. (figure 20).

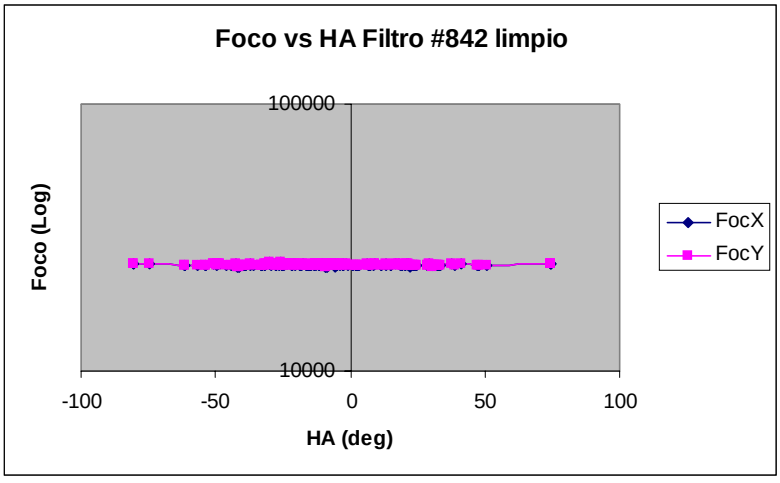
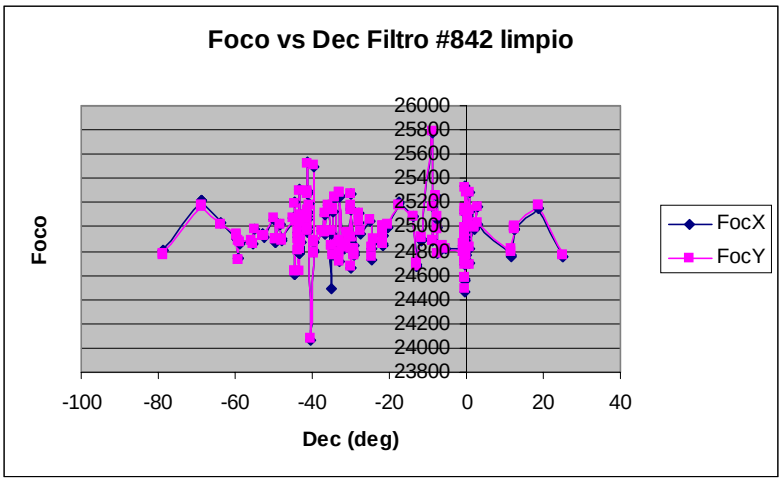
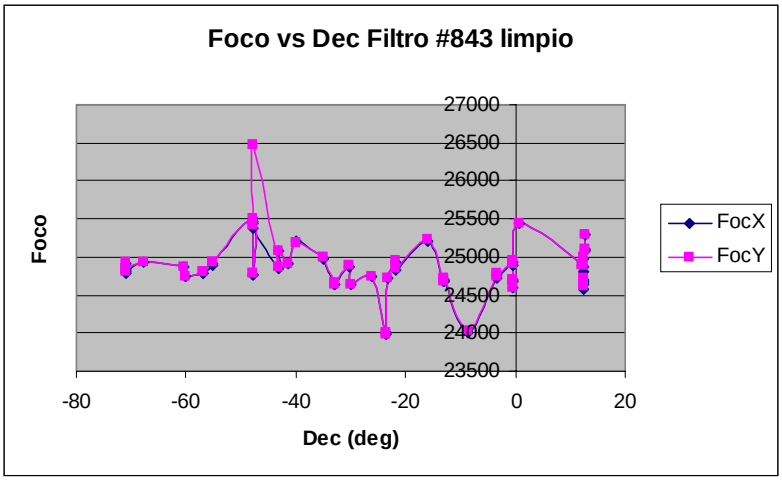


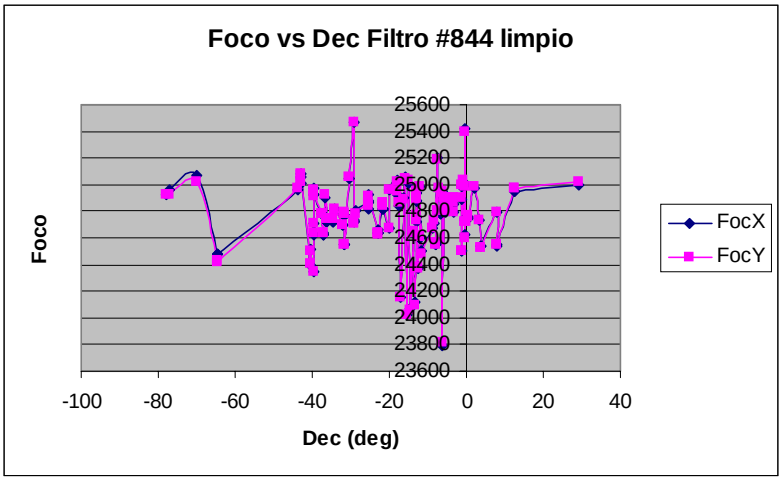
Figure 20

Focus Value against Declination

Id. to the Hour Angle analysis, the dependence of the Focus Value against this variable is not clear, as it can be watched in figures 21 to 24, as the results shows a bad adjustment of the model, although a little better than the past variable.







Figures 21, 22 and 23

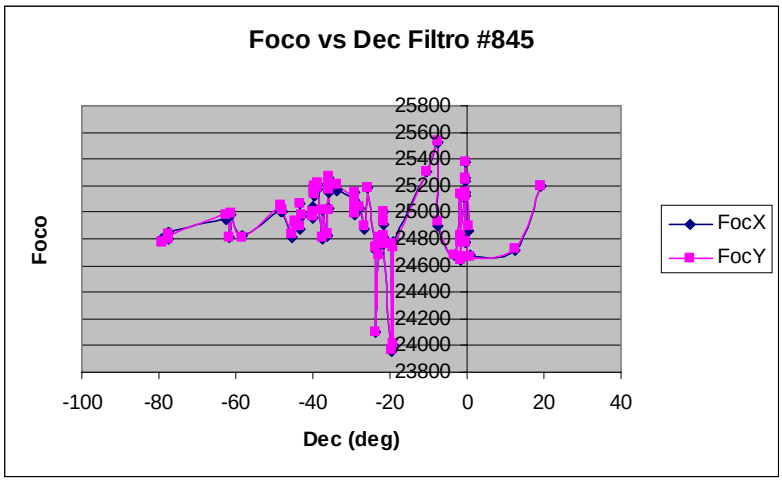


Figure 24

Filter #842:

Note: $\beta=[\beta_0\ \beta_1\ \beta_2]$
mod: $Y=\beta_0 + \beta_1 \cdot X + \beta_2 \cdot X^2$

Focus X	Focus Y
β = [24915 -1 0]	β = [24930 -1 0]
s^2 = 40350	s^2 = 38517
R^2 = 0,0056	R^2 = 0,0036

Filter #843:

Focus X	Focus Y
β = [24748 1 0]	β = [24797 0 0]
s^2 = 84570	s^2 = 120370
R^2 = 0,5569	R^2 = 0,4730

Filter #844:

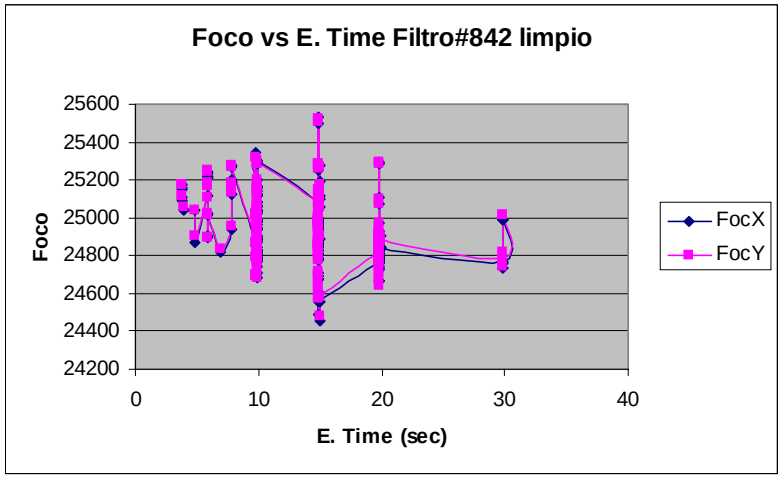
Focus X	Focus Y
β = [24750 2 0]	β = [24755 2 0]
s^2 = 55582	s^2 = 56993
R^2 = 0,3159	R^2 = 0,3078

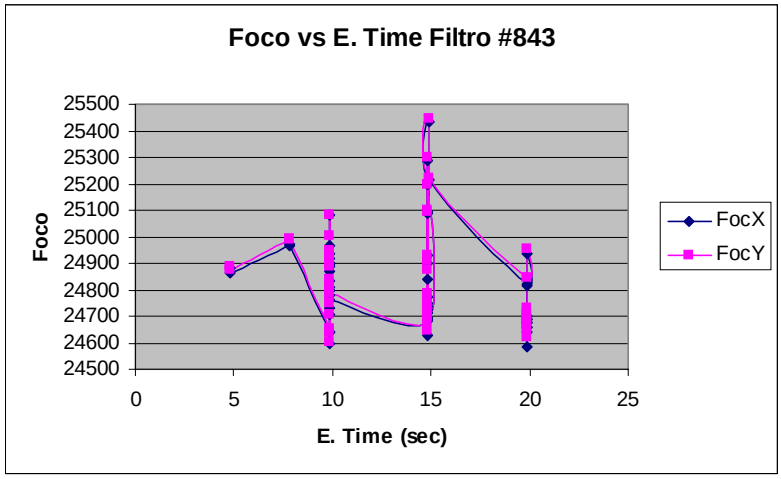
Filter #845:

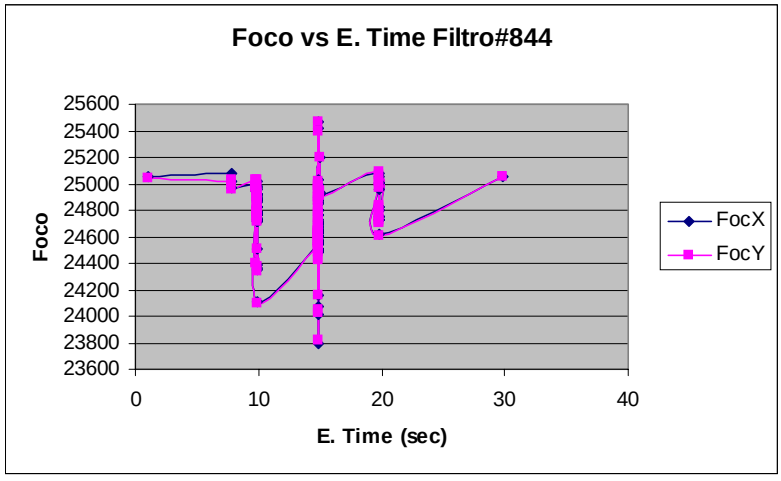
Focus X	Focus Y
β = [24923 -3 0]	β = [24926 –3 0]
s^2 = 59808	s^2 = 60865
R^2 = 0,4138	R^2 = 0,4138

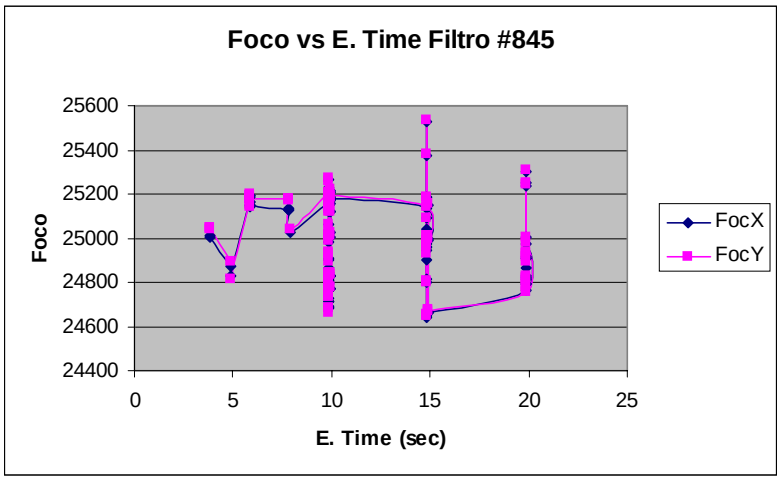
Focus Value against Exposure Time

The decision to analyze this variable as a significant one was taken according to the criteria of its influence in the quantity of light received by the telescope, which can make an effect in the characteristics of the image of one start in the receptor equipment and influence in its quality or its focus determination. However, if it won't be an adequate model, this will appear, in its direct correlation or in the multiple variable analysis.









Figures 22 to 25

Can be observed in the past figures a very little linear dependence, which produces a declination of the Focus Value with the amount of Exposing Time, but this has to be checked with the mathematical results.

Filter #842:

Note: (β=[β_0 β_1])

Focus X	Focus Y
β = [25.108 -13]	β = [25.108 -12]
s^2 = 29577	s^2 = 28142
R^2 = 0,1278	R^2 = 0,12

Filter #843:

Focus X	Focus Y
β = [254940 -8]	β = [24960 -9]
s^2 = 31540	s^2 = 31519
R^2 = 0,7405	R^2 = 0,7411

Filter #844:

Focus X	Focus Y
β = [24779 -2]	β = [24792 -3]
s^2 = 57219	s^2 = 56189
R^2 = 0,2212	R^2 = 0,2187

Filter #845:

Focus X	Focus Y
β = [25056 -6]	β = [25071 -7]
s^2 = 35268	s^2 = 36507
R^2 = 0,5054	R^2 = 0,4978

As can be observed, the relationship is very deficient, but with a better accuracy than was obtained with the Hour Angle and with the Declination.

Chapter 3

Analysis by Added Variables

After observed all the models, it has to be determined the primary variable, which its approximation are the better of all of them. It was chosen the model based in the Long Serrurier Temperature, because it shows the higher value of the Multiple Determination Coefficient (R2). In that case, the Long Serrurier Temperature (LS) will be X1.

To adding a second variable to a multiple model, it has to be evaluated the R coefficient for the mixed models Y(X1,M1), Y(X1,HA), Y(X1,Dec), Y(X1,ET) and to compare with the value of R for the simple model Y(X1). In the cases of the HA and Dec, we will use simple linear models because its coefficient of higher order were zero.

In this way, the double general model form will be:

$$Y = \beta_0 + \beta_1 \cdot X_1 + \beta_2 \cdot X_1 \cdot X_2 + \beta_3 \cdot X_2$$

The results are presented bellow:

Model Y(X1, M1)

Filter #842:

Note: (β=[β_0 β_1 β_2 β_3])

Focus X	Focus Y
β = [25882 –71 –1 –9]	β = [25853 –65 –1 8]
s^2 = 2880,3	s^2 = 2363,1
R^2 = 0,9237	R^2 = 0,9340

Difference R^2(X1, M1) – R^2(X1): 0,9237 – 0,9256 = - 0,0019 (< 0 !!)
Difference R^2(X1, M1) – R^2(X1): 0,9340 – 0,9354 = - 0,0014 (< 0 !!)

Filter #843:

Focus X	Focus Y
β = [25856 –52 0 –22]	β = [25798 –51 –1 –12]
s^2 = 1996,9	s^2 = 2113,5
R^2 = 0,9822	R^2 = 0,9809

Difference R^2(X1, M1) – R^2(X1): 0,9822 – 0,9805 = 0,0017 (> 0)
Difference R^2(X1, M1) – R^2(X1): 0,9809 – 0,9801 = 0,0008 (> 0)

Filter #844:

Focus X	Focus Y
β = [25859 –77 0 +8]	β = [25799 –78 –1 23]
s^2 = 863,71	s^2 = 968,98
R^2 = 0,9851	R^2 = 0,9835

Difference R^2(X1, M1) – R^2(X1): 0,9851 – 0,9852 = –0,0001 (< 0 !!)
Difference R^2(X1, M1) – R^2(X1): 0,9835 – 0,9822 = 0,0013 (> 0)

Filter #845:

Focus X	Focus Y
β = [25692 –59 –1 14]	β = [25631 –59 –2 28]
s^2 = 1743,0	s^2 = 1884,9
R^2 = 0,9799	R^2 = 0,9787

Difference R^2(X1, M1) – R^2(X1): 0,9799 – 0,9798 = 0,0001 (> 0)
Difference R^2(X1, M1) – R^2(X1): 0,9787 – 0,9773 = 0,0014 (> 0)

Model Y(X1, HA)

Filter #842:

Note: (β=[β_0 β_1 β_2 β_3])

Focus X	Focus Y
β = [25935 –73 0 –1]	β = [25930 –72 0 –1]
s^2 = 2846,4	s^2 = 2371,5
R^2 = 0,9245	R^2 = 0,9336

Difference R^2(X1, HA) – R^2(X1): 0,9245 – 0,9256 = - 0,0011 (< 0 !!)
Difference R^2(X1, HA) – R^2(X1): 0,9336 – 0,9354 = - 0,0018 (< 0 !!)

Filter #843:

Focus X	Focus Y
β = [25889 –76 0 –1]	β = [25899 –77 0 2]
s^2 = 2170,5	s^2 = 2049,6
R^2 = 0,9806	R^2 = 0,9814

Difference R^2(X1, HA) – R^2(X1): 0,9806 – 0,9805 = 0,0001 (> 0)
Difference R^2(X1, HA) – R^2(X1): 0,9814 – 0,9801 = 0,0013 (> 0)

Filter #844:

Focus X	Focus Y
β = [25872 –74 0 0]	β = [25882 –74 0 –1]
s^2 = 828,5355	s^2 = 973,9312
R^2 = 0,9854	R^2 = 0,9831

Difference R^2(X1, HA) – R^2(X1): 0,9854 – 0,9852 = 0,0002 (> 0)
Difference R^2(X1, HA) – R^2(X1): 0,9831 – 0,9822 = 0,0009 (> 0)

Filter #845:

Focus X	Focus Y
β = [25813 –67 0 1]	β = [25834 –68 0 0]
s^2 = 1760,6	s^2 = 1991,9
R^2 = 0,9794	R^2 = 0,9771

Difference R^2(X1, HA) – R^2(X1): 0,9794 – 0,9798 = - 0,0004 (< 0!!)
Difference R^2(X1, HA) – R^2(X1): 0,9771 – 0,9773 = - 0,0002 (< 0!!)

Model Y(X1, DEC)

Filter #842:

Note: (β=[β_0 β_1 β_2 β_3])

Focus X	Focus Y
β = [25944 –75 0 0]	β = [25944 –73 0 0]
s^2 = 2755,4	s^2 = 2339,7
R^2 = 0,9269	R^2 = 0,9345

Difference R^2(X1, DEC) – R^2(X1): 0,9269 – 0,9256 = 0,0013 (> 0)
Difference R^2(X1, DEC) – R^2(X1): 0,9345 – 0,9354 = - 0,0009 (< 0 !!)

Filter #843:

Focus X	Focus Y
β = [25915 –78 0 2]	β = [25930 –79 0 3]
s^2 = 2098,6	s^2 = 1934,2
R^2 = 0,9812	R^2 = 0,9825

Difference R^2(X1, DEC) – R^2(X1): 0,9812 – 0,9805 = 0,0007 (> 0)
Difference R^2(X1, DEC) – R^2(X1): 0,9825 – 0,9801 = 0,0024 (> 0)

Filter #844:

Focus X	Focus Y
β = [25852 –73 0 –2]	β = [25870 –73 0 –2]
s^2 = 827,4015	s^2 = 1016, 2
R^2 = 0,9854	R^2 = 0,9824

Difference R^2(X1, DEC) – R^2(X1): 0,9854 – 0,9852 = 0,0002 (> 0)
Difference R^2(X1, DEC) – R^2(X1): 0,9824 – 0,9822 = 0,0002 (> 0)

Filter #845:

Focus X	Focus Y
β = [25813 –67 0 0]	β = [25815 –67 0 –1]
s^2 = 1741,7	s^2 = 1955,6
R^2 = 0,9796	R^2 = 0,9775

Difference R^2(X1, DEC) – R^2(X1): 0,9796 – 0,9798 = - 0,0002 (< 0!!)
Difference R^2(X1, DEC) – R^2(X1): 0,9775 – 0,9773 = 0,0002 (> 0)

Model Y(X1, ET)

Filter #842:

Note: (β=[β_0 β_1 β_2 β_3])

Focus X	Focus Y
β = [26047 –81 1 –7]	β = [25999 –77 0 –4]
s^2 = 2867,0	s^2 = 2380,2
R^2 = 0,9239	R^2 = 0,9334

Difference R^2(X1, ET) – R^2(X1): 0,9239 – 0,9256 = - 0,0017 (< 0 !!)
Difference R^2(X1, ET) – R^2(X1): 0,9334 – 0,9354 = - 0,0020 (< 0 !!)

Filter #843:

Focus X	Focus Y
β = [25869 –74 0 –1]	β = [25919 –74 0 –1]
s^2 = 2276,9	s^2 = 2231,9
R^2 = 0,9556	R^2 = 0,9550

Difference R^2(X1, ET) – R^2(X1): 0,9556 – 0,9805 = - 0,0249 (< 0!!)
Difference R^2(X1, ET) – R^2(X1): 0,9550 – 0,9801 = - 0,0251 (< 0!!)

Filter #844:

Focus X	Focus Y
β = [25974 –80 0 –6]	β = [25866 –71 0 2]
s^2 = 847,9760	s^2 = 1009,9
R^2 = 0,9751	R^2 = 0,9711

Difference R^2(X1, ET) – R^2(X1): 0,9751 – 0,9852 = - 0,0101 (< 0!!)
Difference R^2(X1, ET) – R^2(X1): 0,9711 – 0,9822 = - 0,0111 (< 0!!)

Filter #845:

Focus X	Focus Y
β = [25883 –70 0 –7]	β = [25932 –73 0 –9]
s^2 = 1663,4	s^2 = 1778,0
R^2 = 0,9692	R^2 = 0,9680

Difference R^2(X1, ET) – R^2(X1): 0,9692 – 0,9798 = - 0,0106 (< 0!!)
Difference R^2(X1, ET) – R^2(X1): 0,9680 – 0,9773 = - 0,0093 (< 0!!)

Conclusions

According to the analysis, we have five preliminary estimation models of the Focus Value. With the Long Serrurier Temperatures (LS) and the Main Mirror Temperature (M1) as work variables in four of them, and only with the LS variable in the first one; all to be evaluated in its accuracy and practical error margins. These models are:

Filter #842: (ESO842)

$F_X = 25947 - 74 \cdot LS$ accuracy: 93.6%
$F_Y = 25936 - 72 \cdot LS$ accuracy: 94.5%

Filter #843: (ESO843)

$F_X = 25856 - 52 \cdot LS - 22 \cdot M1$ accuracy: 98.2%
$F_Y = 25798 - 51 \cdot LS - LS \cdot M1 - 12 \cdot M1$ accuracy: 98.1%

Filter #844: (ESO844)

$F_X = 25859 - 77 \cdot LS + 8 \cdot M1$ accuracy: 98.5%
$F_Y = 25799 - 78 \cdot LS - LS \cdot M1 + 23 \cdot M1$ accuracy: 98.4%

Filter #845: (ESO845)

$F_X = 25692 - 59 \cdot LS - LS \cdot M1 + 14 \cdot M1$ accuracy: 98.0%
$F_Y = 25631 - 59 \cdot LS - 2 \cdot LS \cdot M1 + 28 \cdot M1$ accuracy: 97.9%

It is recommended, anywise, to destine a complete night of observation time to check these equations and to get several more values for the database, to verify if it is possible to include variables related to the Hour Angle and the Declination.